

Time: 3 hour

Max. Marks: 80

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1 Attempt All questions

A If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ then find the eigen values of A^2 5

B Find Laplace transform of $f(t) = te^t \cos 2t$ 5

C Find the half range Sine Series for $f(x) = x$, where $x \in (-\pi, \pi)$ 5

D Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. 5

Q2

A Using Green's theorem in a plane to evaluate the line integral 6

$$\oint_C (xy + y^2)dx + x^2dy$$

Around the boundary of the region defined by $y=x^2$ and $y=x$

B Find the Eigen values and Eigen vectors of the matrix 6

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

C Show that the function $v = e^x \sin y$ satisfies Laplace's equation, also find analytic function. 8

Q3

A Prove that $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational. 6

B Find the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ 6

C Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 8

where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

12

Q4

A Using Stokes theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ 6
 Where $\vec{F} = x^2\vec{i} + xy\vec{j}$ where C is the boundary of the rectangle
 $x=0, y=0, x=a, y=b$

B

Evaluate $\int_0^\infty \frac{e^{-t} \sin 2t}{t} dt$, using Laplace transforms 6

C

Using Convolution theorem find $L^{-1} \left[\frac{s^2}{(s^2+1)(s^2+4)} \right]$ 8

Q5

A Find $L \{ e^{-4t} \sin 3t \cos 2t \}$ 6

B Prove that the vector field \vec{F} on R^3 defined by
 $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + (3xz^2 + 2)\vec{k}$
 is conservative and find its scalar potential. 6

C Find the Fourier Series for $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in $0 \leq x \leq 2\pi$ 8

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in $(-2, 2)$ 6

B Prove that the matrix A is diagonalisable, find the transforming matrix and the diagonal matrix. 6

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

C

i) Find $L^{-1} \left\{ \log \left(\frac{s^2+1}{s^2+2} \right) \right\}$ 4

ii) Find $L^{-1} \left\{ \frac{1}{s^2+2s+10} \right\}$ 4